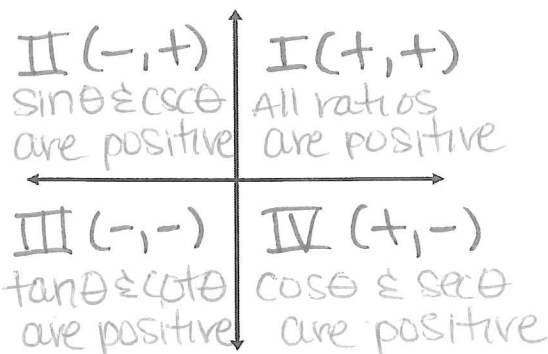


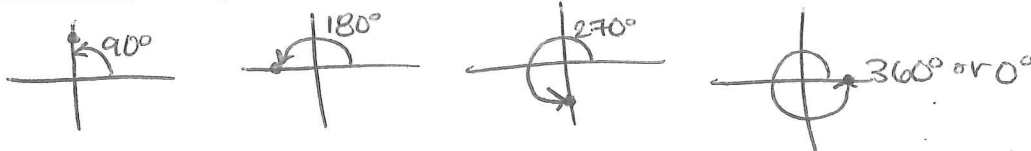
13-3 Notes: Trigonometric Functions of General Angles (Day 3 – Quadrantal & Radians)

In general, the following will always be true for any problem you are asked to find a trig function:

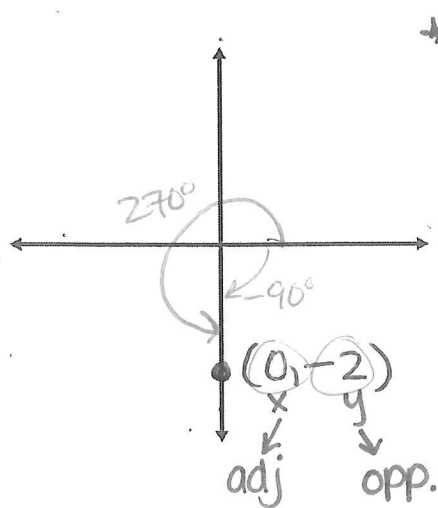


* be careful when labeling sides of Δ *

Quadrantal Angles: an angle whose terminal side falls on one of the axes (i. e. $0^\circ, 90^\circ, 180^\circ, 270^\circ$)



Example 1: Find the values of the six trig functions of θ if the terminal side contains the point (0, -2).



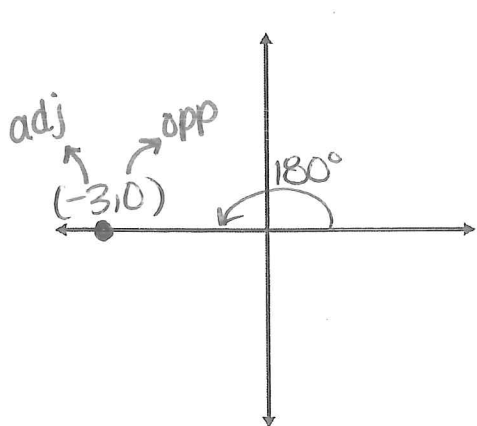
* Find hypotenuse using $a^2 + b^2 = c^2$ *

$$0^2 + (-2)^2 = (\text{hyp})^2 \quad \sin \theta = \frac{0}{4} = \frac{-2}{2} = \boxed{-1} \quad \csc \theta = \frac{2}{-2} = \boxed{-1}$$

$$0 + 4 = (\text{hyp})^2 \quad \cos \theta = \frac{0}{4} = \frac{0}{2} = \boxed{0} \quad \sec \theta = \frac{2}{0} = \boxed{\text{undefined}}$$

$$4 = (\text{hyp})^2 \quad \tan \theta = \frac{0}{4} = \frac{-2}{0} = \boxed{\text{undefined}} \quad \cot \theta = \frac{0}{-2} = \boxed{0}$$

Example 2: Find the values of the six trig functions of θ if the terminal side contains the point (-3, 0).



$$(-3)^2 + 0^2 = (\text{hyp})^2$$

$$9 + 0 = (\text{hyp})^2$$

$$9 = (\text{hyp})^2$$

$$3 = \text{hyp}$$

$$\sin \theta = \frac{0}{9} = \frac{0}{3} = \boxed{0} \quad \csc \theta = \frac{3}{0} = \boxed{\text{undefined}}$$

$$\cos \theta = \frac{9}{9} = \frac{-3}{3} = \boxed{-1} \quad \sec \theta = \frac{3}{-3} = \boxed{-1}$$

$$\tan \theta = \frac{0}{9} = \frac{0}{-3} = \boxed{0} \quad \cot \theta = \frac{-3}{0} = \boxed{\text{undefined}}$$

Recall: Convert the following degree measures to radians \Rightarrow divide by 180°

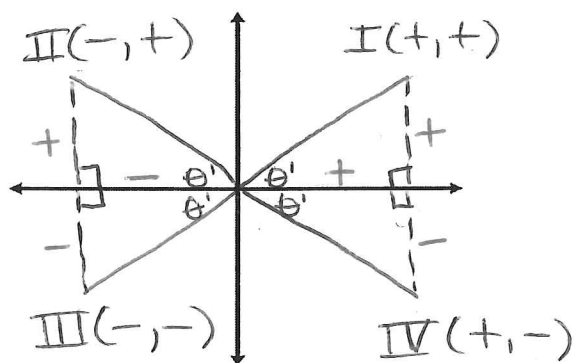
$$30^\circ = \frac{\pi}{6}$$

$$45^\circ = \frac{\pi}{4}$$

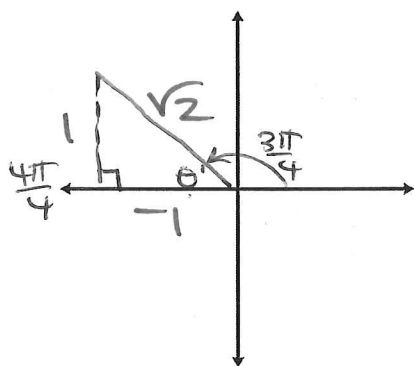
$$60^\circ = \frac{\pi}{3}$$

Reminders when drawing angles and triangles to find exact values:

- ALWAYS draw the triangle to the x-axis
- The radius is the hypotenuse and is always POSITIVE
- θ' , which represents the reference angle, is always to the x-axis and is always positive



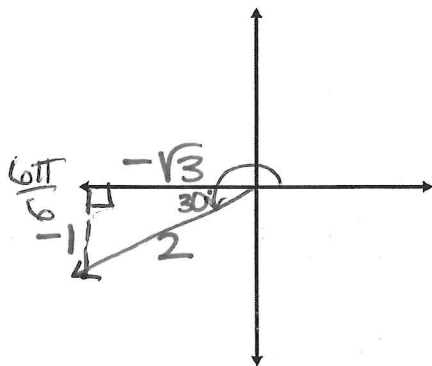
Example 3: Use a reference angle to find the exact value of $\cos \frac{3\pi}{4}$.



$$\theta' = \frac{\pi}{4} = 45^\circ$$

$$\begin{aligned} \cos \theta &= \frac{A}{H} = \frac{-1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} \\ &= \boxed{-\frac{\sqrt{2}}{2}} \end{aligned}$$

Example 4: Use a reference angle to find the exact value of $\sec \frac{7\pi}{6}$.



$$\theta' = \frac{\pi}{6} = 30^\circ$$

$$\begin{aligned} \sec \theta &= \frac{H}{A} = \frac{2 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} \\ &= \boxed{-\frac{2\sqrt{3}}{3}} \end{aligned}$$

13-3 Exact Values

Find the exact value of each trigonometric function.

1) $\csc 150^\circ$

2) $\tan 135^\circ$

3) $\sin 150^\circ$

4) $\cot 0^\circ$

5) $\tan 210^\circ$

6) $\cos 30^\circ$

7) $\sin 330^\circ$

8) $\sin 150^\circ$

9) $\tan -90^\circ$

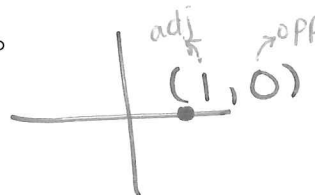


$$\tan \theta = \frac{o}{a}$$

$$= \frac{-1}{0}$$

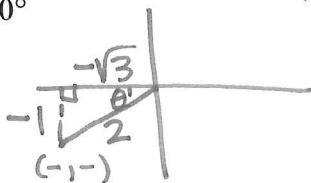
$$\boxed{\text{undefined}}$$

10) $\tan 0^\circ$



11) $\cos 210^\circ$

$\theta' = 30^\circ$



$$\cos \theta = \frac{a}{h}$$

$$= \frac{-\sqrt{3}}{2}$$

$$\boxed{\frac{-\sqrt{3}}{2}}$$

12) $\sin 300^\circ$

13) $\sec 930^\circ$

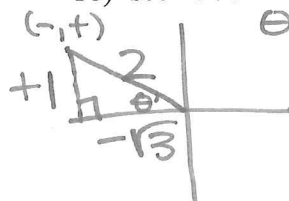
14) $\tan 960^\circ$

15) $\cos -390^\circ$

16) $\tan -180^\circ$

17) $\csc 510^\circ$

18) $\sec -570^\circ$



$$\sec \theta = \frac{h}{a}$$

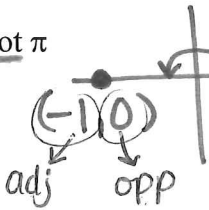
$$= \frac{2 \cdot \sqrt{3}}{-\sqrt{3} \cdot \sqrt{3}}$$

$$= \boxed{-\frac{2\sqrt{3}}{3}}$$

19) $\tan \frac{\pi}{3}$

20) $\cos \frac{2\pi}{3}$

21) $\cot \pi$



Find hyp:

$$(-1)^2 + 0^2 = (\text{hyp})^2$$

$$1 = \text{hyp}$$

$$\cot \theta = \frac{A}{O} = \frac{-1}{0}$$

undefined

22) $\cos \frac{\pi}{4}$

23) $\csc \frac{2\pi}{3}$

24) $\tan \frac{7\pi}{4}$

25) $\tan -\frac{\pi}{2}$

26) $\cos -\frac{\pi}{4}$

27) $\csc -\frac{3\pi}{4}$

28) $\cos \frac{5\pi}{3}$

29) $\tan \frac{4\pi}{3}$

30) $\cot -\frac{\pi}{6}$

31) $\cot -\frac{4\pi}{3}$

★ 32) $\tan -\frac{16\pi}{3}$

$\theta' = \frac{\pi}{3} = 60^\circ$

$\tan \theta = \frac{O}{A} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

$\sqrt{3}$

33) $\cos -\frac{10\pi}{3}$

34) $\cos -\frac{8\pi}{3}$

35) $\sec -\frac{8\pi}{3}$

36) $\cot \frac{9\pi}{4}$

Answers to 13-3 Exact Values

1) 2

2) -1

3) $\frac{1}{2}$

4) Undefined

5) $\frac{\sqrt{3}}{3}$

6) $\frac{\sqrt{3}}{2}$

7) $-\frac{1}{2}$

8) $\frac{1}{2}$

9) Undefined

10) 0

11) $-\frac{\sqrt{3}}{2}$

12) $-\frac{\sqrt{3}}{2}$

13) $-\frac{2\sqrt{3}}{3}$

14) $\sqrt{3}$

15) $\frac{\sqrt{3}}{2}$

16) 0

17) 2

18) $-\frac{2\sqrt{3}}{3}$

19) $\sqrt{3}$

20) $-\frac{1}{2}$

21) Undefined

22) $\frac{\sqrt{2}}{2}$

23) $\frac{2\sqrt{3}}{3}$

24) -1

25) Undefined

26) $\frac{\sqrt{2}}{2}$

27) $-\sqrt{2}$

28) $\frac{1}{2}$

29) $\sqrt{3}$

30) $-\sqrt{3}$

31) $-\frac{\sqrt{3}}{3}$

32) $-\sqrt{3}$

33) $-\frac{1}{2}$

34) $-\frac{1}{2}$

35) -2

36) 1