13-3 Notes (Day 1), Trigonometric Functions of General Angles

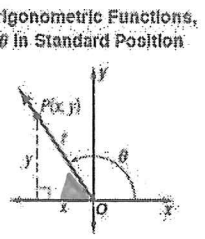
Objective: To find exact values of trigonometric functions for general & quadrantal angles

Recall the six trigonometric functions of an angle θ SOH CAH TOA

$\sin\theta = \frac{opp}{hyp}$ $\quad \csc\theta = \frac{hyp}{opp}$

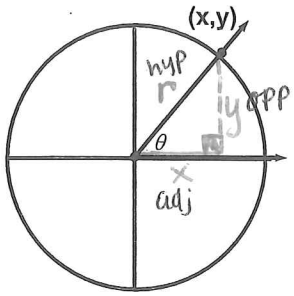
$\cos\theta = \frac{adj}{hyp}$ $\quad \sec\theta = \frac{hyp}{adj}$

$\tan\theta = \frac{opp}{adj}$ $\quad \cot\theta = \frac{adj}{opp}$



△ drawn to x-axis & uses reference θ'*

When point (x,y) is on the terminal side of angle θ, in a general circle, the trig functions are defined as:

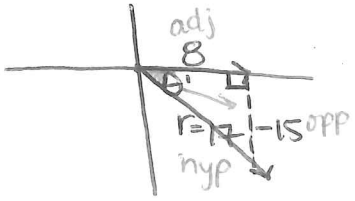


$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$

Ex 1 Find the exact values of the six trig functions of θ if the terminal side contains (8,-15)



Find r using $a^2+b^2=c^2$

$8^2+(-15)^2=r^2$

$64+225=r^2$

$\sqrt{289}=\sqrt{r^2}$

$r=17$

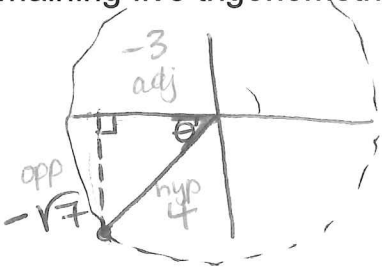
radius/hypotenuse is ALWAYS positive

$\sin\theta = \frac{-15}{17}$ $\quad \csc\theta = \frac{17}{-15} = -\frac{17}{15}$

$\cos\theta = \frac{8}{17}$ $\quad \sec\theta = \frac{17}{8}$

$\tan\theta = \frac{-15}{8}$ $\quad \cot\theta = -\frac{8}{15}$

Example 2: Suppose θ is an angle in standard position whose terminal side is in Quadrant III and $\sec\theta = -\frac{4}{3}$. Find the exact values of the remaining five trigonometric functions of θ



* find opp side using $a^2+b^2=c^2$

$(-3)^2+b^2=(4)^2$

$9+b^2=16$

$b^2=7$

$b=\sqrt{7}$

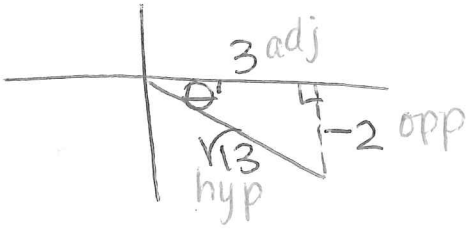
$\sin\theta = \frac{-\sqrt{7}}{4}$ $\quad \csc\theta = \frac{-4\cdot\sqrt{7}}{\sqrt{7}\cdot\sqrt{7}} = \frac{-4\sqrt{7}}{7}$

$\cos\theta = \frac{-3}{4}$ $\quad \sec\theta = \frac{-4}{3}$

$\tan\theta = \frac{-\sqrt{7}}{3}$ $\quad \cot\theta = \frac{3\cdot\sqrt{7}}{\sqrt{7}\cdot\sqrt{7}} = \frac{3\sqrt{7}}{7}$

$= \frac{\sqrt{7}}{3}$

Example 3: Suppose θ is an angle in standard position whose terminal side is in Quadrant IV and $\tan\theta = -\frac{2}{3}$. Find the exact values of the remaining five trigonometric functions of θ



* find hyp using $a^2+b^2=c^2$

$(-2)^2+3^2=r^2$

$4+9=r^2$

$\sqrt{13}=r$

$\sin\theta = \frac{-2\cdot\sqrt{13}}{\sqrt{13}\cdot\sqrt{13}} = \frac{-2\sqrt{13}}{13}$ $\quad \csc\theta = \frac{\sqrt{13}}{-2}$

$\cos\theta = \frac{3\cdot\sqrt{13}}{\sqrt{13}\cdot\sqrt{13}} = \frac{3\sqrt{13}}{13}$ $\quad \sec\theta = \frac{\sqrt{13}}{3}$

$\tan\theta = \frac{-2}{3}$ $\quad \cot\theta = \frac{-3}{2}$