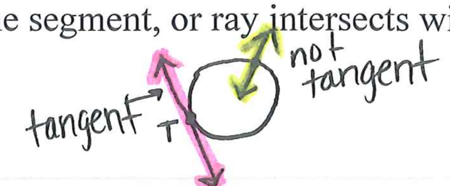
Name: *Key*

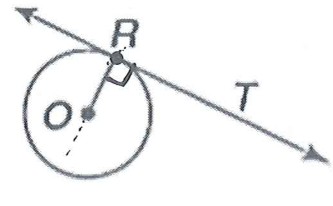
10-5 Tangent Notes

A line, line segment, or ray that intersects a circle in exactly one point is the tangent. The point that the line, line segment, or ray intersects with the circle is called the point of tangency.

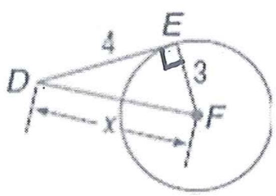


If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

Example: If $\overleftrightarrow{RT}$ is a tangent, $\overline{OR} \perp \overleftrightarrow{RT}$.



Example 1: $\overline{ED}$ is tangent to Circle F at point E. Find x.



$DE \perp EF$ (tangent & radius)

$(DE)^2 + (EF)^2 = (DF)^2$

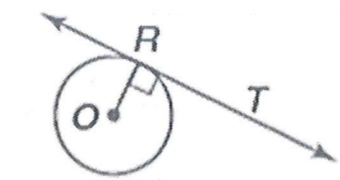
$4^2 + 3^2 = (DF)^2$

$\sqrt{25} = \sqrt{x^2}$

$\boxed{x = 5}$

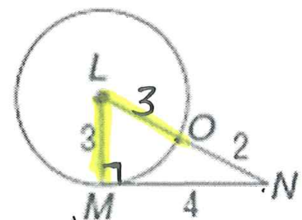
If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

Example: If $\overline{OR} \perp \overleftrightarrow{RT}$, $\overleftrightarrow{RT}$ is a tangent.



Example 2:

a) Determine whether $\overline{MN}$ is tangent to Circle L. Justify your reasoning.



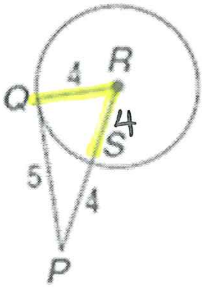
(show MN is $\perp$ LM using $a^2+b^2=c^2$)

$(LM)^2 + (MN)^2 \stackrel{?}{=} (LN)^2$

$3^2 + 4^2 \stackrel{?}{=} 5^2$

$25 = 25$ ✓

$\therefore$ $\overline{MN}$ is tangent to circle L



b) Determine whether $\overline{PQ}$ is tangent to Circle R. Justify your reasoning.

$(RQ)^2 + (QP)^2 \stackrel{?}{=} (RP)^2$

$4^2 + 5^2 \stackrel{?}{=} 8^2$

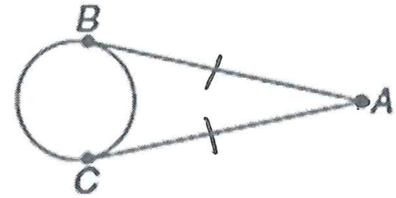
$41 \neq 64$

$\therefore$ $\overline{PQ}$ is not tangent to circle R.

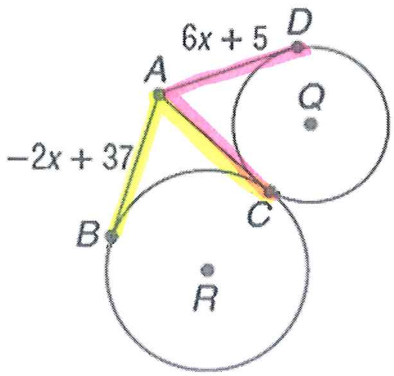
Congruent Tangents

If two segments from the same exterior point are tangent to a circle, then they are congruent.

Example: $\overline{AB} \cong \overline{AC}$



Example 3: Find *x*. Assume that segments that appear tangent to circles are tangent.



$AB \cong AC$ → from same exterior point and both are tangent

$AC \cong AD$ → same as above

$\therefore AB \cong AD \Rightarrow AB = AD$

$-2x + 37 = 6x + 5$

$-8x = -32$

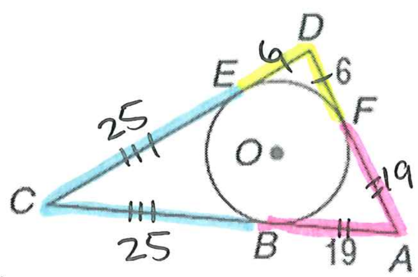
$\boxed{x = 4}$

Circumscribed Polygons

Polygons can also be circumscribed about a circle, or the circle is inscribed in the polygon.

The vertices of the polygon DO NOT lie on the circle, but every side of the polygon is tangent to the circle.

Example 4: Triangle *ADC* is circumscribed about Circle O. Find the perimeter of Triangle *ADC* if $\overline{EC} = \overline{DE} + \overline{AF}$.



$DF \cong DE$ $AF \cong AB$

$DE = 6$ $AF = 19$

$EC \cong BC$

$EC = DE + AF$

$EC = 6 + 19 = 25$

$P = 6 + 6 + 19 + 19 + 25 + 25$

$\boxed{P = 100 \text{ units}}$

intersects in 2 points

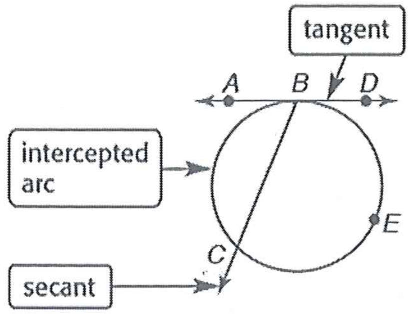
exactly one point on circle

THEOREM 10.13

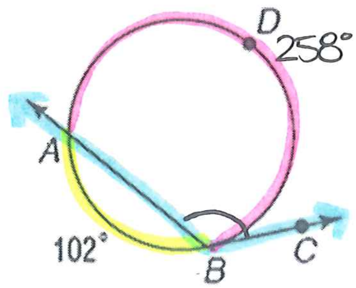
If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

Examples: $m\angle ABC = \frac{1}{2}m\widehat{BC}$

$m\angle DBC = \frac{1}{2}m\widehat{BEC}$



Example 1: Find $m\angle ABC$ if $m\widehat{AB} = 102$



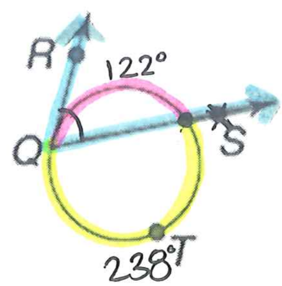
$\boxed{m\widehat{ADB}} = 360° - m\widehat{AB}$

$= 360° - 102° = 258°$

$\angle ABC = \frac{1}{2}(m\widehat{ADB})$

$= \frac{1}{2}(258°) = \boxed{129°}$

Example 2: Find $m\angle RQS$ if $m\widehat{QTS} = 238$



$m\widehat{QS} = 360° - m\widehat{QTS}$

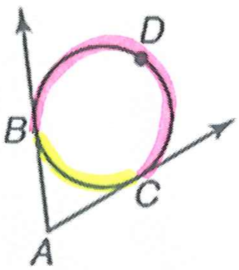
$= 360° - 238°$

$m\widehat{QS} = 122°$

$\angle RQS = \frac{1}{2}(m\widehat{QS})$

$= \frac{1}{2}(122°) = \boxed{61°}$

Two Tangents

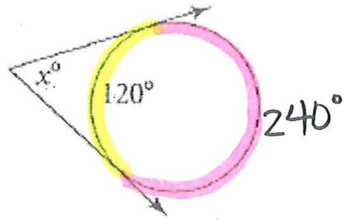


$m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$

$m\angle A = \frac{1}{2}(\underset{\text{arc}}{\text{outer}} - \underset{\text{arc}}{\text{inner}})$

Example 3:

$360° - 120° = \text{outer arc}$
$= 240°$



$x° = \frac{1}{2}(\text{outer} - \text{inner})$

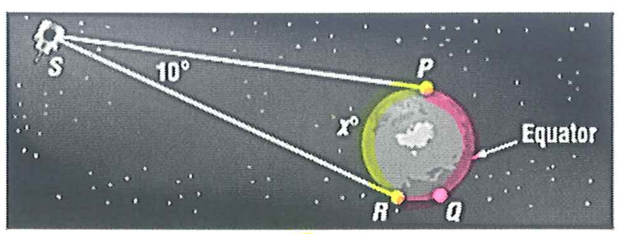
$x° = \frac{1}{2}(240° - 120°)$

$x° = \frac{1}{2}(120°)$

$\boxed{x° = 60°}$

Example 4:

SATELLITES Suppose a satellite *S* orbits above Earth rotating so that it appears to hover directly over the equator. Use the figure to determine the arc measure on the equator visible to this satellite.



$m\widehat{PR} = \boxed{x°} = \text{inner}$
$m\widehat{PQR} = \boxed{360° - x} = \text{outer}$

$\angle S = \frac{1}{2}(\underset{\text{outer}}{m\widehat{PQR}} - \underset{\text{inner}}{m\widehat{PR}})$

$10° = \frac{1}{2}(\underline{360° - x} - \underline{x})$

$10° = \frac{1}{2}(360° - 2x)$

$10 = 180° - 1x$

$\boxed{x = 170°}$