Name: ______________________________ Circles – Day 3

ARCS AND CHORDS NOTES (10.3)

THEOREM 10.2

In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

Abbreviations:

In ⊙, 2 minor arcs are ≅, corr. chords are ≅.

In ⊙, 2 chords are ≅, corr. minor arcs are ≅.

Examples:

If $\overline{AB} \cong \overline{CD}$, $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

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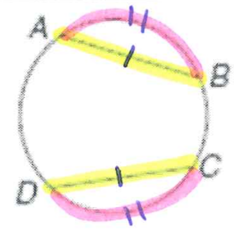
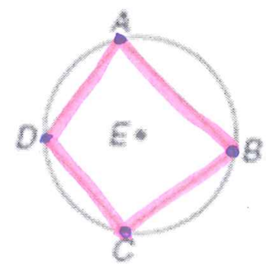


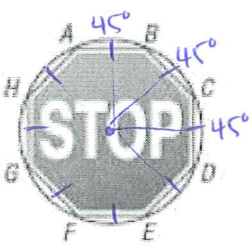
The chords of adjacent arcs can form a polygon.

Quadrilateral ABCD is an inscribed polygon because all of its vertices lie on the circle.

Circle E is circumscribed about the polygon because it contains all the vertices of the polygon.



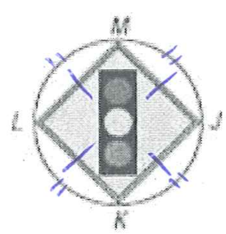
Let's see some examples:



octagon
all ≅ sides

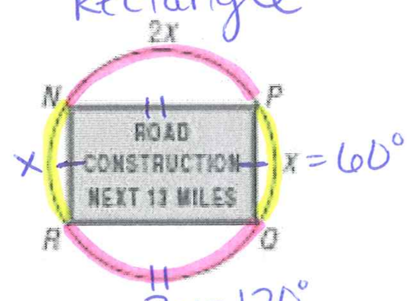
$\frac{360°}{8} = \boxed{45°}$

square



$\frac{360°}{4} = \boxed{90°}$

Rectangle

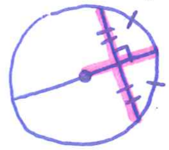


$2x + 2x + x + x = 360°$

$6x = 360°$

$x = 60°$

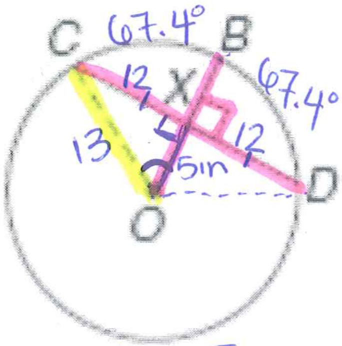
In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.



∗ Chord is bisected NOT RADIUS

Example 1: Given the information below, find CX, OX, XB, and the $m\widehat{CD}$.

Circle O has a radius of 13 inches. Radius $\overline{OB}$ is perpendicular to chord $\overline{CD}$, which is 24 inches long.



$\cos\theta = \frac{5}{13}$

$\theta = \cos^{-1}(5/13)$

$\angle O = 67.4°$

CX = $\frac{1}{2}(CD) = \frac{1}{2}(24) = 12$in

OX = 5 in

$(OX)^2 + 12^2 = 13^2$

$(OX)^2 + 144 = 169$

$(OX)^2 = 25$

XB = 8 in

$OX + XB = OB$

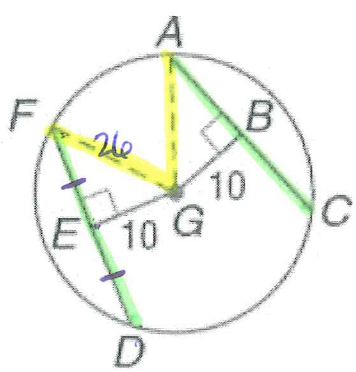
$5 + XB = 13$

$m\widehat{CD} = m\widehat{CB} + m\widehat{BD}$

$= 67.4° + 67.4° = \boxed{134.8°}$

In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

Example 2: Chords $\overline{AC}$ *and* $\overline{DF}$ are equidistant from the center. If the radius of Circle G is 26, find AC and DE.



$FE^2 + EG^2 = FG^2$

$FE^2 + 10^2 = 26^2$

$FE^2 + 100 = 676$

$FE^2 = 576$

$FE = 24$

$FE \cong DE$

$\boxed{DE = 24}$

$FD = 2(FE)$

$FD = 2(24)$

$FD = 48$

$AC \cong FD$

$\boxed{AC = 48}$