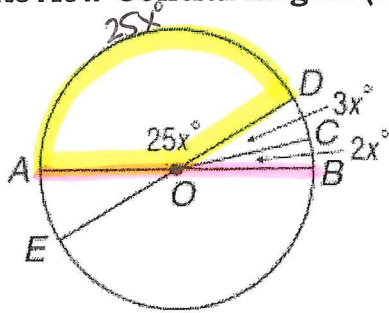


# Inscribed Angles and Arcs

**Review Central Angles (with algebra)** Example 1: Find the  $m\angle AOD$ .



$$\angle AOB = 180^\circ \text{ (semicircle)}$$

$$\angle AOD + \angle DOB + \angle COB = 180^\circ$$

$$25x + 3x + 2x = 180^\circ$$

$$30x = 180^\circ$$

$$x = 6$$

$$\begin{aligned} \angle AOD &= 25x \\ &= 25(6) \end{aligned}$$

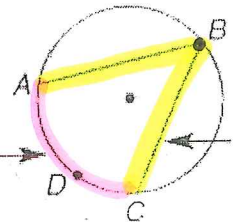
$$\boxed{\angle AOD = 150^\circ}$$

## Inscribed Angles

An inscribed angle is an angle whose vertex is

ON the circle and its sides are chords

ADC is the arc intercepted by  $\angle ABC$ .



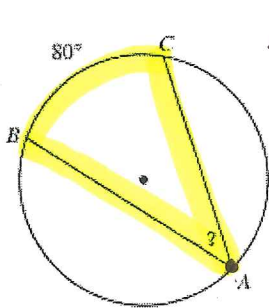
Inscribed angle is  $\frac{1}{2}$  its arc.  
 $\text{inscribed } \angle = \frac{1}{2}(\text{arc})$

OR

Arc intercepted is  $\frac{2 \text{ times}}$  the inscribed angle  
 $\text{arc} = 2(\text{inscribed } \angle)$

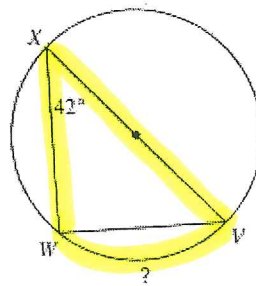
### Example 1

Find the measure of the arc or angle indicated.



$$\begin{aligned} \angle BAC &= \frac{1}{2}(\widehat{BC}) \\ &= \frac{1}{2}(80^\circ) \end{aligned}$$

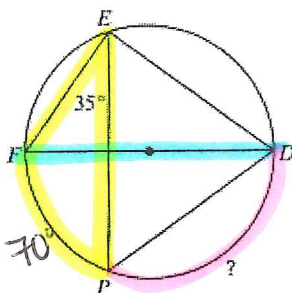
$$\boxed{\angle BAC = 40^\circ}$$



$$\widehat{WV} = 2(\angle XWV)$$

$$\widehat{WV} = 2(42^\circ)$$

$$\boxed{\widehat{WV} = 84^\circ}$$

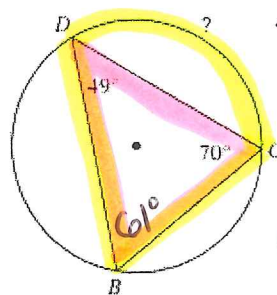


$$\widehat{FP} = 2(\angle FEP)$$

$$\widehat{FP} = 2(35^\circ) = 70^\circ$$

$$\begin{aligned} \widehat{FP} + \widehat{PD} &= 180^\circ \\ &\text{(semicircle)} \end{aligned}$$

$$\boxed{\widehat{PD} = 110^\circ}$$



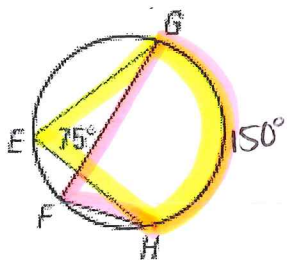
$$\begin{aligned} \angle D + \angle C + \angle B &= 180^\circ \\ &\text{(\Delta Sum)} \end{aligned}$$

$$\angle B = 61^\circ$$

$$\widehat{DC} = 2(\angle B)$$

$$\boxed{\widehat{DC} = 2(61^\circ) = 122^\circ}$$

Example 2 What is the  $m \angle F$ ?



$$\angle E = 75^\circ$$

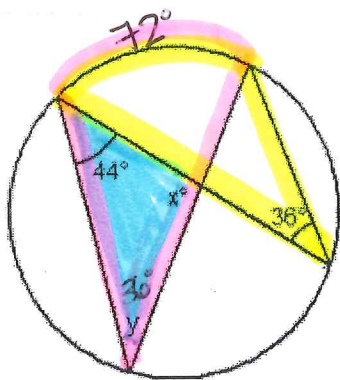
$$\widehat{GH} = 2(\angle E) = 2(75^\circ) = 150^\circ$$

$$\angle F = \frac{1}{2}(\widehat{GH}) = \frac{1}{2}(150^\circ) = \boxed{75^\circ}$$

What did you notice about  $\angle E$  and  $\angle F$ ? they are the same /  $\cong$   
 \*have same arc\*

Why? This is because if two inscribed angles of a circle intercept the **same arc** or **congruent arcs**, then the  $\angle$ s are  $\cong$ .

Example 3: Find  $x$  and  $y$ .



$$y = \frac{1}{2}(72^\circ) = \boxed{36^\circ}$$

$$x + 44^\circ + 36^\circ = 180^\circ$$

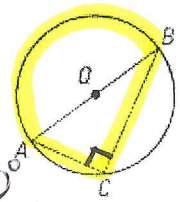
( $\Delta$  Sum)

$$\boxed{x = 100^\circ}$$

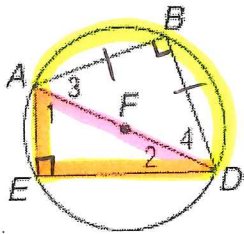
## Angles of Inscribed Polygons

If the inscribed angle intercepts a semicircle, the angle is a right angle.

$$\angle ACB = \frac{1}{2}(\widehat{AB}) = \frac{1}{2}(180^\circ) = 90^\circ$$



Example 4: Triangles ABD and ADE are inscribed in Circle F with  $\widehat{AB} \cong \widehat{BD}$ . Find the measures of  $\angle 1$  and  $\angle 2$  if  $m\angle 1 = 12x - 8$  and  $m\angle 2 = 3x + 8$ .



$$\angle 1 + \angle 2 + 90^\circ = 180^\circ \quad (\Delta \text{ Sum})$$

$$\angle E = 90^\circ \quad \text{b/c intercepts a semicircle}$$

$$12x - 8 + 3x + 8 + 90^\circ = 180^\circ$$

$$15x = 90$$

$$\underline{x = 6}$$

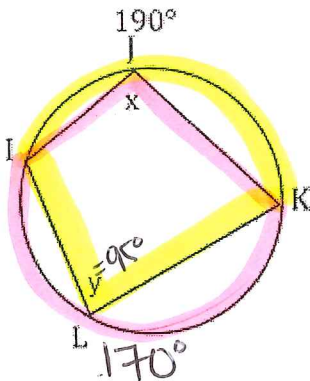
$$\angle 1 = 12(6) - 8 = \boxed{64^\circ}$$

$$\angle 2 = 3(6) + 8 = \boxed{26^\circ}$$

Example 5: Quadrilateral IJKL is inscribed in Circle P.

If  $m\widehat{JK} = 190^\circ$ , find  $x$  and  $y$ .

↳ all points on circle



$$\angle ILK = \frac{1}{2}(\widehat{JK}) = \frac{1}{2}(190^\circ) = 95^\circ$$

$$\boxed{y = 95^\circ}$$

$$\angle IJK = \frac{1}{2}(\widehat{ILK}) = \frac{1}{2}(170^\circ) = 85^\circ$$

$$\boxed{x = 85^\circ}$$

$$x + y = 180^\circ \Rightarrow x \text{ \& \; } y \text{ are supplementary}$$

If a quadrilateral is inscribed in a circle, then its opposite  $\angle$ s  
are supplementary (add =  $180^\circ$ )