NAME _____ DATE _____ PERIOD ____

# 9-4 Study Guide and Intervention

## Common Logarithms

**Common Logarithms** Base 10 logarithms are called **common logarithms**. The expression $\log_{10} x$ is usually written without the subscript as $\log x$. Use the LOG key on your calculator to evaluate common logarithms.

The relation between exponents and logarithms gives the following identity.

| Inverse Property of Logarithms and Exponents | $10^{\log x} = x$ |
|---|---|

**Example 1** **Evaluate $\log 50$ to four decimal places.**

LOG 50 ≈ 1.698970004 ≈ 1.6990

**Example 2** **Solve $3^{2x+1} = 12$.** ⇒ change forms

(base, exp)

$\log_3 12 = 2x + 1$

changed to common logs: $\frac{\log 12}{\log 3} = 2x + 1$

$2.261859507 = 2x + 1$ solve for x

$1.261859507 = 2x$

$.6309 \approx x$

**Exercises**

**Use a calculator to evaluate each expression to four decimal places.**

1. log 18
2. log 39
3. log 120
4. log 5.8
5. log 42.3
6. log 0.003

**Solve each equation or inequality. Round to four decimal places.**

7. $4^{3x} = 12$
8. $6^{x+2} = 18$
9. $5^{4x-2} = 120$
10. $7^{3x-1} \geq 21$
11. $2.4^{x+4} = 30$
12. $6.5^{2x} \geq 200$
13. $3.6^{4x-1} = 85.4$
14. $2^{x+5} = 3^{x-2}$
15. $9^{3x} = 4^{5x+2}$
16. $6^{x-5} = 2^{7x+3}$

NAME _____ DATE _____ PERIOD ____

# 9-4 Study Guide and Intervention *(continued)*

## Common Logarithms

**Change of Base Formula** The following formula is used to change expressions with different logarithmic bases to common logarithm expressions.

| | |
|---|---|
| **Change of Base Formula** | For all positive numbers $a$, $b$, and $n$, where $a \neq 1$ and $b \neq 1$, $\log_a n = \frac{\log_b n}{\log_b a}$ |

**Example** **Express $\log_8 15$ in terms of common logarithms. Then approximate its value to four decimal places.**

change to base 10 log

$$\log_8 15 = \frac{\log 15}{\log 8} \approx 1.3023$$

**Exercises**

**Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.**

1. $\log_3 16$
2. $\log_2 40$
3. $\log_5 35$
4. $\log_4 22$
5. $\log_{12} 200$
6. $\log_2 50$
7. $\log_5 0.4$
8. $\log_3 2$
9. $\log_4 28.5$
10. $\log_3 (20)^2$
11. $\log_6 (5)^4$
12. $\log_8 (4)^5$
13. $\log_5 (8)^3$
14. $\log_2 (3.6)^6$
15. $\log_{12} (10.5)^4$

$$\frac{\log (10.5)^4}{\log 12} \approx 3.7851$$

NAME _____ DATE _____ PERIOD ____

# 9-5 Study Guide and Intervention

## Base e and Natural Logarithms

**Base e and Natural Logarithms** The irrational number $e \approx 2.71828\ldots$ often occurs as the base for exponential and logarithmic functions that describe real-world phenomena.

| | |
|---|---|
| **Natural Base e** | As $n$ increases, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx 2.71828\ldots$. <br> $\ln x = \log_e x$ |

The functions $y = e^x$ and $y = \ln x$ are inverse functions.

| | |
|---|---|
| **Inverse Property of Base e and Natural Logarithms** | $e^{\ln x} = x$ $\quad \ln e^x = x$ |

Natural base expressions can be evaluated using the $e^x$ and ln keys on your calculator.

**Example 1** **Evaluate ln 1685.**

LN 1685 ≈ 7.4295

**Example 2** **Write a logarithmic equation equivalent to $e^{2x} = 7$.**

exp, base

$\log_e 7 = 2x \Rightarrow \ln 7 = 2x$

**Example 3** **Evaluate $\ln e^{18}$.**

$\log_e e^{18} = 18$

same base ⇒ cancel

**Exercises**

**Use a calculator to evaluate each expression to four decimal places.**

1. ln 732
2. ln 84,350
3. ln 0.735
4. ln 100
5. ln 0.0824
6. ln 2.388
7. ln 128,245
8. ln 0.00614

**Write an equivalent exponential or logarithmic equation.**

9. $e^{15} = x$
10. $e^{3x} = 45$
11. $\ln 20 = x$
12. $\ln x = 8$
13. $e^{-5x} = 0.2$
14. $\ln (4x) = 9.6$
15. $e^{8.2} = 10x$
16. $\ln 0.0002 = x$

**Evaluate each expression.**

17. $\ln e^3$
18. $e^{\ln 42}$
19. $e^{\ln 0.5}$
20. $\ln e^{16.2}$

NAME _____ DATE _____ PERIOD ____

# 9-5 Study Guide and Intervention *(continued)*

## Base e and Natural Logarithms

**Equations and Inequalities with e and ln** All properties of logarithms from earlier lessons can be used to solve equations and inequalities with natural logarithms.

**Example** **Solve each equation or inequality.**

**a.** $3e^{2x} + 2 = 10$ (*like example 5c in notes)

isolate

$3e^{2x} = 8$

$e^{2x} = 8/3 \Rightarrow$ change forms $\Rightarrow$ $\log_e 8/3 = 2x$

$\ln 8/3 = 2x$

$\frac{\ln(8/3)}{2} = x$

$.4904 \approx x$

**b.** $\ln(4x - 1) < 2$

$\log_e(4x-1) < 2 \Rightarrow$ change forms

base, exp

$e^2 < 4x - 1$ solve for x

$\frac{e^2+1}{4} < x$

$2.0973 < x$

* $e^{\wedge}$ 2nd LN

**Exercises**

**Solve each equation or inequality.**

1. $e^{4x} = 120$
2. $e^x \leq 25$
3. $e^{x-2} + 4 = 21$
4. $\ln 6x \geq 4$
5. $\ln(x + 3) - 5 = -2$
6. $e^{-8x} \leq 50$
7. $e^{4x-1} - 3 = 12$
8. $\ln(5x + 3) = 3.6$
9. $2e^{3x} + 5 = 2$
10. $6 + 3e^{x+1} = 21$
11. $\ln(2x - 5) = 8$