NAME ______________________________ DATE ____________ PERIOD _____

8-1 Study Guide and Intervention

Multiplying and Dividing Rational Expressions

Simplify Rational Expressions A ratio of two polynomial expressions is a **rational expression.** To simplify a rational expression, divide both the numerator and the denominator by their greatest common factor (GCF).

Multiplying Rational Expressions	For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, if $b \neq 0$ and $d \neq 0$.
Dividing Rational Expressions	For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$, if $b \neq 0$, $c \neq 0$, and $d \neq 0$.

Example **Simplify each expression.**

a. $\frac{24a^5b^2}{(2ab)^4}$

$$\frac{24a^5b^2}{(2ab)^4} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 3 \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{a}} \cdot a \cdot \overset{1}{\cancel{b}} \cdot \overset{1}{\cancel{b}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{a}} \cdot \underset{1}{\cancel{b}} \cdot \underset{1}{\cancel{b}} \cdot b \cdot b} = \frac{3a}{2b^2}$$

b. $\frac{3r^2s^3}{5t^4} \cdot \frac{20t^2}{9r^3s}$

$$\frac{3r^2s^3}{5t^4} \cdot \frac{20t^2}{9r^3s} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{r}} \cdot \overset{1}{\cancel{r}} \cdot \overset{1}{\cancel{s}} \cdot s \cdot s \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{t}} \cdot \overset{1}{\cancel{t}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{t}} \cdot \underset{1}{\cancel{t}} \cdot t \cdot t \cdot \underset{1}{\cancel{3}} \cdot 3 \cdot \underset{1}{\cancel{r}} \cdot \underset{1}{\cancel{r}} \cdot r \cdot \underset{1}{\cancel{s}}} = \frac{2 \cdot 2 \cdot s \cdot s}{3 \cdot r \cdot t \cdot t} = \frac{4s^2}{3rt^2}$$

c. $\frac{x^2 + 8x + 16}{2x - 2} \div \frac{x^2 + 2x - 8}{x - 1}$

$$\frac{x^2 + 8x + 16}{2x - 2} \div \frac{x^2 + 2x - 8}{x - 1} = \frac{x^2 + 8x + 16}{2x - 2} \cdot \frac{x - 1}{x^2 + 2x - 8}$$

$$= \frac{\overset{1}{\cancel{(x + 4)}}(x + 4)\overset{1}{\cancel{(x - 1)}}}{2\underset{1}{\cancel{(x - 1)}}(x - 2)\underset{1}{\cancel{(x + 4)}}} = \frac{x + 4}{2(x - 2)}$$

Exercises

Simplify each expression.

1. $\frac{(-2ab^2)^3}{20ab^4}$

2. $\frac{4x^2 - 12x + 9}{9 - 6x}$

3. $\frac{x^2 + x - 6}{x^2 - 6x - 27}$

4. $\frac{3m^3 - 3m}{6m^4} \cdot \frac{4m^5}{m + 1}$

5. $\frac{c^2 - 3c}{c^2 - 25} \cdot \frac{c^2 + 4c - 5}{c^2 - 4c + 3}$

6. $\frac{(m - 3)^2}{m^2 - 6m + 9} \cdot \frac{m^3 - 9m}{m^2 - 9}$

7. $\frac{6xy^4}{25z^3} \div \frac{18xz^2}{5y}$

8. $\frac{16p^2 - 8p + 1}{14p^4} \div \frac{4p^2 + 7p - 2}{7p^5}$

9. $\frac{2m - 1}{m^2 - 3m - 10} \div \frac{4m^2 - 1}{4m + 8}$

8-1 Study Guide and Intervention *(continued)*

Multiplying and Dividing Rational Expressions

Simplify Complex Fractions A **complex fraction** is a rational expression whose numerator and/or denominator contains a rational expression. To simplify a complex fraction, first rewrite it as a division problem.

Example **Simplify** $\dfrac{\frac{3s-1}{s}}{\frac{3s^2+8s-3}{s^4}}$.

$$\frac{\frac{3s-1}{s}}{\frac{3s^2+8s-3}{s^4}} = \frac{3s-1}{s} \div \frac{3s^2+8s-3}{s^4}$$ Express as a division problem.

$$= \frac{3s-1}{s} \cdot \frac{s^4}{3s^2+8s-3}$$ Multiply by the reciprocal of the divisor.

$$= \frac{\overset{1}{\cancel{(3s-1)}}\overset{s^3}{\cancel{s^4}}}{\underset{1}{\cancel{s}}\underset{1}{\cancel{(3s-1)}}(s+3)}$$ Factor.

$$= \frac{s^3}{s+3}$$ Simplify.

Exercises

Simplify.

1. $\dfrac{\frac{x^3y^2z}{a^2b^2}}{\frac{a^3x^2y}{b^2}}$

2. $\dfrac{\frac{a^2bc^3}{x^2y^2}}{\frac{ab^2}{c^4x^2y}}$

3. $\dfrac{\frac{b^2-1}{3b+2}}{\frac{b+1}{3b^2-b-2}}$

4. $\dfrac{\frac{b^2-100}{b^2}}{\frac{3b^2-31b+10}{2b}}$

5. $\dfrac{\frac{x-4}{x^2+6x+9}}{\frac{x^2-2x-8}{3+x}}$

6. $\dfrac{\frac{a^2-16}{a+2}}{\frac{a^2+3a-4}{a^2+a-2}}$

7. $\dfrac{\frac{2x^2+9x+9}{x+1}}{\frac{10x^2+19x+6}{5x^2+7x+2}}$

8. $\dfrac{\frac{b+2}{b^2-6b+8}}{\frac{b^2+b-2}{b^2-16}}$

9. $\dfrac{\frac{x^2-x-2}{x^3+6x^2-x-30}}{\frac{x+1}{x+3}}$

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8-1 Study Guide and Intervention

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Example Simplify each expression.

a. $\frac{24a^5b^2}{(2ab)^4}$

$$\frac{24a^5b^2}{(2ab)^4} = \frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b}{2 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b} = \frac{3a}{2b^2}$$

b. $\frac{3r^2s^3}{5t^4} \cdot \frac{20t^2}{9r^3s}$

$$\frac{3r^2s^3}{5t^4} \cdot \frac{20t^2}{9r^3s} = \frac{3 \cdot r \cdot r \cdot s \cdot s \cdot s \cdot 2 \cdot 2 \cdot 5 \cdot t \cdot t}{5 \cdot t \cdot t \cdot t \cdot t \cdot 3 \cdot 3 \cdot r \cdot r \cdot r \cdot s} = \frac{2 \cdot 2 \cdot s \cdot s}{3 \cdot r \cdot t \cdot t} = \frac{4s^2}{3rt^2}$$

c. $\frac{x^2 + 8x + 16}{2x - 2} \div \frac{x^2 + 2x - 8}{x - 1}$

$$\frac{x^2 + 8x + 16}{2x - 2} \div \frac{x^2 + 2x - 8}{x - 1} = \frac{x^2 + 8x + 16}{2x - 2} \cdot \frac{x - 1}{x^2 + 2x - 8}$$

$$= \frac{(x + 4)(x + 4)(x - 1)}{2(x - 1)(x - 2)(x + 4)} = \frac{x + 4}{2(x - 2)}$$

Exercises

Simplify each expression.

1. $\frac{(-2ab^2)^3}{20ab^4}$ $-\frac{2a^2b^2}{5}$

2. $\frac{4x^2 - 12x + 9}{9 - 6x}$ $\frac{3 - 2x}{3}$

3. $\frac{x^2 + x - 6}{x^2 - 6x - 27}$ $\frac{x - 2}{x - 9}$

4. $\frac{3m^3 - 3m}{6m^4} \cdot \frac{4m^5}{m + 1}$ $2m^2(m - 1)$

5. $\frac{c^2 - 3c}{c^2 - 25} \cdot \frac{c^2 + 4c - 5}{c^2 - 4c + 3}$ $\frac{c}{c - 5}$

6. $\frac{(m - 3)^2}{m^2 - 6m + 9} \cdot \frac{m^3 - 9m}{m^2 - 9}$ m

7. $\frac{6xy^4}{25z^3} \div \frac{18xz^2}{5y}$ $\frac{y^5}{15z^5}$

8. $\frac{16p^2 - 8p + 1}{14p^4} \div \frac{4p^2 + 7p - 2}{7p^5}$ $\frac{p(4p - 1)}{2(p + 2)}$

9. $\frac{2m - 1}{m^2 - 3m - 10} \div \frac{4m^2 - 1}{4m + 8}$ $\frac{4}{(2m + 1)(m - 5)}$

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8-1 Study Guide and Intervention *(continued)*

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Simplify Complex Fractions A **complex fraction** is a rational expression whose numerator and/or denominator contains a rational expression. To simplify a complex fraction, first rewrite it as a division problem.

Example Simplify $\frac{\frac{3s - 1}{s}}{\frac{3s^2 + 8s - 3}{s^4}}$.

$$\frac{\frac{3s - 1}{s}}{\frac{3s^2 + 8s - 3}{s^4}} = \frac{3s - 1}{s} \div \frac{3s^2 + 8s - 3}{s^4}$$ Express as a division problem.

$$= \frac{3s - 1}{s} \cdot \frac{s^4}{3s^2 + 8s - 3}$$ Multiply by the reciprocal of the divisor.

$$= \frac{(3s - 1)s^4}{s(3s - 1)(s + 3)}$$ Factor.

$$= \frac{s^3}{s + 3}$$ Simplify.

Exercises

Simplify.

1. $\frac{\frac{x^3y^2z}{a^2b^2}}{\frac{a^3x^2y}{b^2}}$ $\frac{xyz}{a^5}$

2. $\frac{\frac{a^2bc^3}{x^2y^2}}{\frac{ab^2}{c^4x^2y}}$ $\frac{ac^7}{by}$

3. $\frac{\frac{b^2 - 1}{3b + 2}}{\frac{b + 1}{3b^2 - b - 2}}$ $(b - 1)^2$

4. $\frac{\frac{b^2 - 100}{b^2}}{\frac{3b^2 - 31b + 10}{2b}}$ $\frac{2(b + 10)}{b(3b - 1)}$

5. $\frac{\frac{x - 4}{x^2 + 6x + 9}}{\frac{x^2 - 2x - 8}{3 + x}}$ $\frac{1}{(x + 3)(x + 2)}$

6. $\frac{\frac{a^2 - 16}{a + 2}}{\frac{a^2 + 3a - 4}{a^2 + a - 2}}$ $a - 4$

7. $\frac{\frac{2x^2 + 9x + 9}{x + 1}}{\frac{10x^2 + 19x + 6}{5x^2 + 7x + 2}}$ $x + 3$

8. $\frac{\frac{b + 2}{b^2 - 6b + 8}}{\frac{b^2 + b - 2}{b^2 - 16}}$ $\frac{b + 4}{(b - 1)(b - 2)}$

9. $\frac{\frac{x^2 - x - 2}{x^3 + 6x^2 - x - 30}}{\frac{x + 1}{x + 3}}$ $\frac{1}{x + 5}$